# Midterm Exam: MAT 203

Instructions: Complete all problems below. You may not use calculators or other aides, including cell phones and books. Show all of your work. **Be sure to write your name and student ID on each page that you hand in.**

1.(25pts) Consider the line $L$ given by $\mathbf{r}(t) = (2t-3, t+4, -3t-1)$.

a) Find a unit vector $\mathbf{v}$ parallel to this line.

b) Find the equation for the plane perpendicular to $\mathbf{v}$ and passing through $(1,1,-3)$.

c) Find the distance from the origin $(0,0,0)$ to line $L$.

a) $r'(t) = 2\vec{i} + \vec{j} - 3\vec{k}$ , $\|r'(t)\| = \sqrt{14}$

$$\boxed{v = \frac{r'(t)}{\|r'(t)\|} = \frac{2\vec{i} + \vec{j} - 3\vec{k}}{\sqrt{14}}}$$

b) $\boxed{2(x-1) + (y-1) - 3(z+3) = 0}$

c) $r(0) = -3\vec{i} + 4\vec{j} - \vec{k}$

$$\boxed{\text{Dist} = \frac{\| r(0) \times v \|}{\|v\|} = 11 \cdot \sqrt{\frac{3}{14}}}$$

$$r(0) \times v = \frac{-11}{\sqrt{14}} (\vec{i} + \vec{j} + \vec{k})$$

2.(25pts) Consider the surface in $\mathbb{R}^3$ described by the equation

$$x^2 + 4y^2 - z^2 - 2x + 2z - 1 = 0.$$

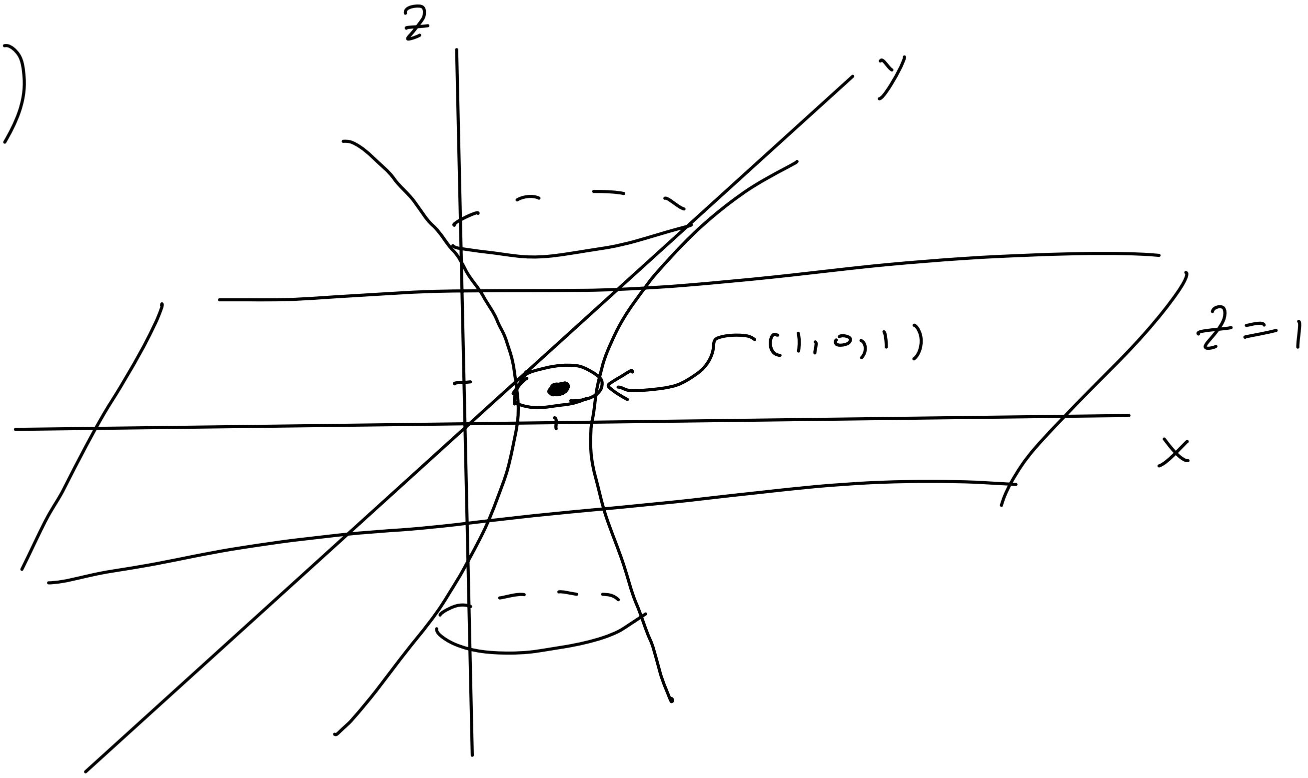
a) Is this surface an ellipsoid, a one-sheeted hyperboloid, a two-sheeted hyperboloid, or a paraboloid? Justify your answer, by rewriting the equation in standard form.

b) Sketch this surface. Make sure to label the salient features of your drawing. For example, if it is centered at a particular point then label that point, and if it is a surface of revolution then label the axis about which it is rotated, etc.

a) $(x^2 - 2x + 1) - 1 + 4y^2 - (z^2 - 2z + 1 - 1) - 1 = 0$

$$\boxed{\begin{gathered}(x-1)^2 + 4y^2 - (z-1)^2 = 1 \\ \text{one-sheeted hyperboloid}\end{gathered}}$$

b)



3.(25pts) An object moves in $\mathbb{R}^3$ with acceleration $\mathbf{a}(t) = 2t\mathbf{i} + e^{-t}\mathbf{j} - (\cos t)\mathbf{k}$.

a) Determine the object's position and velocity vectors at any time $t$.

b) If the object is at rest when $t = 0$, what is its velocity when $t = \pi$?

a) $\boxed{v(t) = t^2 \vec{i} - (e^{-t} - 1)\vec{j} - \sin t \, \vec{k} + v(0)}$

$$\boxed{r(t) = \frac{t^3}{3} \vec{i} + (e^{-t} + t - 1)\vec{j} + (\cos t - 1)\vec{k} + v(0)t + r(0)}$$

b) At rest initially means $v(0) = \vec{0}$.

$$\boxed{v(\pi) = \pi^2 \vec{i} - (e^{-\pi} - 1)\vec{j}}$$

4.(25pts) Consider the function $f(x,y) = \cos\left(\frac{xy}{x^2+2y^2}\right)$.

a) Find the domain and range of $f$.

b) Determine whether $f$ is continuous at the origin, if its value at this point is set to $f(0,0) = 1$. Fully justify your answer.

c) Compute the partial derivative $\frac{\partial f}{\partial x}$.

a) $\boxed{\begin{array}{l} \text{Domain} = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; (x,y) \neq (0,0) \right\} \\ \text{Range} = [-1, 1] \end{array}}$

b) $$\lim_{\substack{x \to 0 \\ y = ax}} \frac{xy}{x^2+2y^2} = \lim_{x \to 0} \frac{ax^2}{(1+2a^2)x^2} = \frac{a}{1+2a^2}$$

Since this takes different values for different choices of $a$, we conclude that

$\boxed{f \text{ is not continuous at the origin.}}$

c) $$\boxed{\frac{\partial f}{\partial x} = \frac{x^2y - y^3}{(x^2+y^2)^2} \cdot \sin\left(\frac{xy}{x^2+2y^2}\right)}.$$